8.2 Use Properties of Parallelograms

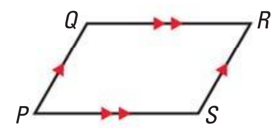
Before You used a property of polygons to find angle measures.

Now You will find angle and side measures in parallelograms.

Why? So you can solve a problem about airplanes, as in Ex. 38.

Key Vocabulary
- **parallelogram**

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. The term "parallelogram *PQRS*" can be written as ▱*PQRS*. In ▱*PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.



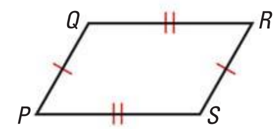
THEOREMS — *For Your Notebook*

THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If *PQRS* is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Proof: p. 516

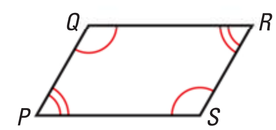


THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

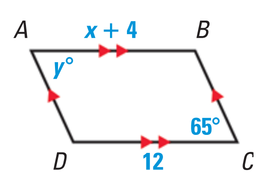
If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof: Ex. 42, p. 520



EXAMPLE 1 Use properties of parallelograms

xy ALGEBRA **Find the values of *x* and *y*.**



ABCD is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of *x*.

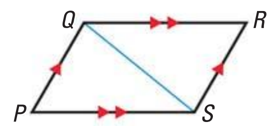
$AB = CD$	**Opposite sides of a ▱ are ≅.**
$x + 4 = 12$	**Substitute $x + 4$ for *AB* and 12 for *CD*.**
$x = 8$	**Subtract 4 from each side.**

By Theorem 8.4, $\angle A \cong \angle C$, or $m\angle A = m\angle C$. So, $y° = 65°$.

▶ In ▱*ABCD*, $x = 8$ and $y = 65$.

PROOF Theorem 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.



GIVEN ▶ *PQRS* is a parallelogram.

PROVE ▶ $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$

Plan for Proof

a. Draw diagonal $\overline{QS}$ to form $\triangle PQS$ and $\triangle RSQ$.

b. Use the ASA Congruence Postulate to show that $\triangle PQS \cong \triangle RSQ$.

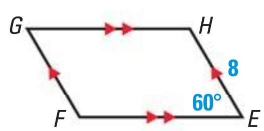
c. Use congruent triangles to show that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Plan in Action

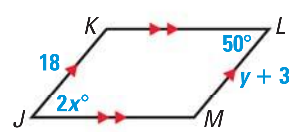
	STATEMENTS	REASONS
a.	1. *PQRS* is a ▱.	1. Given
	2. Draw $\overline{QS}$.	2. Through any 2 points there exists exactly 1 line.
	3. $\overline{PQ} \parallel \overline{RS}$, $\overline{QR} \parallel \overline{PS}$	3. Definition of parallelogram
b.	4. $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$	4. Alternate Interior Angles Theorem
	5. $\overline{QS} \cong \overline{QS}$	5. Reflexive Property of Congruence
	6. $\triangle PQS \cong \triangle RSQ$	6. ASA Congruence Postulate
c.	7. $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$	7. Corresp. parts of ≅ ⚠ are ≅.

✓ GUIDED PRACTICE for Example 1

1. Find *FG* and $m\angle G$.

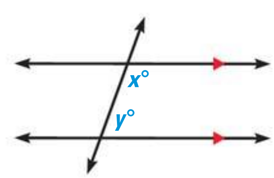


2. Find the values of *x* and *y*.



INTERIOR ANGLES The Consecutive Interior Angles Theorem (page 155) states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.

A pair of consecutive angles in a parallelogram are like a pair of consecutive interior angles between parallel lines. This similarity suggests Theorem 8.5.



$x° + y° = 180°$

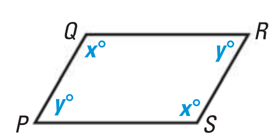
THEOREM *For Your Notebook*

THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If *PQRS* is a parallelogram, then $x° + y° = 180°$.

Proof: Ex. 43, p. 520



EXAMPLE 2 Use properties of a parallelogram

DESK LAMP As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.

Solution

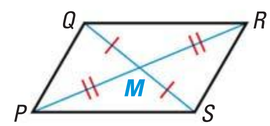
By Theorem 8.5, the consecutive angle pairs in $\square ABCD$ are supplementary. So, $m\angle ADC + m\angle BCD = 180°$. Because $m\angle ADC = 110°$, $m\angle BCD = 180° - 110° = 70°$.

THEOREM — *For Your Notebook*

THEOREM 8.6

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Proof: Ex. 44, p. 521



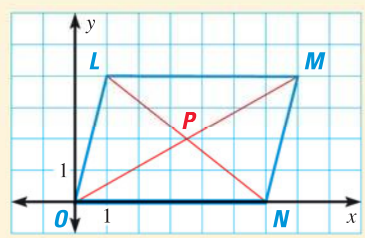
$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$

★ EXAMPLE 3 Standardized Test Practice

The diagonals of $\square LMNO$ intersect at point P. What are the coordinates of P?

Ⓐ $\left(\frac{7}{2}, 2\right)$　　Ⓑ $\left(2, \frac{7}{2}\right)$

Ⓒ $\left(\frac{5}{2}, 2\right)$　　Ⓓ $\left(2, \frac{5}{2}\right)$



SIMPLIFY CALCULATIONS

In Example 3, you can use either diagonal to find the coordinates of P. Using $\overline{OM}$ simplifies calculations because one endpoint is (0, 0).

Solution

By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, P is the midpoint of diagonals $\overline{LN}$ and $\overline{OM}$. Use the Midpoint Formula.

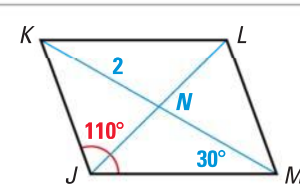
Coordinates of midpoint P of $\overline{OM} = \left(\frac{7+0}{2}, \frac{4+0}{2}\right) = \left(\frac{7}{2}, 2\right)$

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 2 and 3

Find the indicated measure in $\square JKLM$.

3. NM
4. KM
5. $m\angle JML$
6. $m\angle KML$



8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 29, 35, and 41

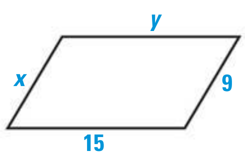
SKILL PRACTICE

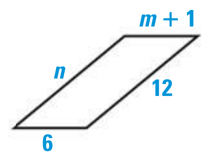
1. **VOCABULARY** What property of a parallelogram is included in the definition of a parallelogram? What properties are described by the theorems in this lesson?

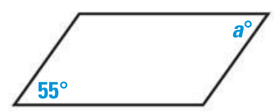
2. ★ **WRITING** In parallelogram *ABCD*, $m\angle A = 65°$. *Explain* how you would find the other angle measures of ▱*ABCD*.

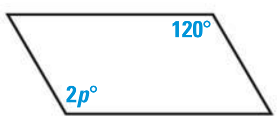
EXAMPLE 1 on p. 515 for Exs. 3–8

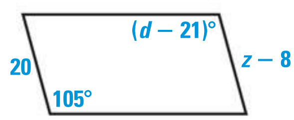
ALGEBRA **Find the value of each variable in the parallelogram.**

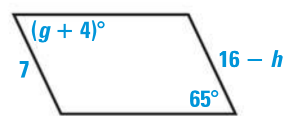
3.


4.


5.


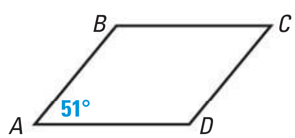
6.


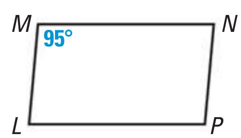
7.


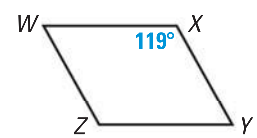
8.


EXAMPLE 2 on p. 517 for Exs. 9–12

FINDING ANGLE MEASURES **Find the measure of the indicated angle in the parallelogram.**

9. Find $m\angle B$.


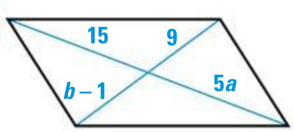
10. Find $m\angle L$.


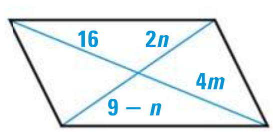
11. Find $m\angle Y$.


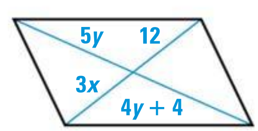
12. **SKETCHING** In ▱*PQRS*, $m\angle R$ is 24 degrees more than $m\angle S$. Sketch ▱*PQRS*. Find the measure of each interior angle. Then label each angle with its measure.

EXAMPLE 3 on p. 517 for Exs. 13–16

ALGEBRA **Find the value of each variable in the parallelogram.**

13.


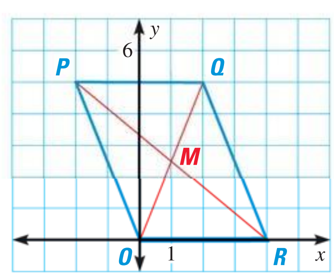
14.


15.


16. ★ **MULTIPLE CHOICE** The diagonals of parallelogram *OPQR* intersect at point *M*. What are the coordinates of point *M*?

Ⓐ $\left(1, \frac{5}{2}\right)$ Ⓑ $\left(2, \frac{5}{2}\right)$

Ⓒ $\left(1, \frac{3}{2}\right)$ Ⓓ $\left(2, \frac{3}{2}\right)$

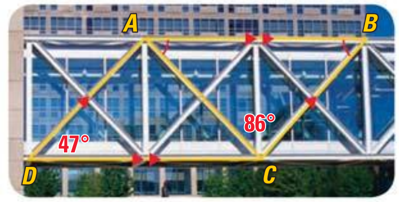


REASONING **Use the photo to copy and complete the statement. *Explain.***

17. $\overline{AD} \cong$ ___?___ **18.** $\angle DAB \cong$ ___?___

19. $\angle BCA \cong$ ___?___ **20.** $m\angle ABC =$ ___?___

21. $m\angle CAB =$ ___?___ **22.** $m\angle CAD =$ ___?___

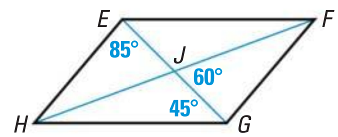


USING A DIAGRAM **Find the indicated measure in ▱*EFGH*. *Explain.***

23. $m\angle EJF$ **24.** $m\angle EGF$

25. $m\angle HFG$ **26.** $m\angle GEF$

27. $m\angle HGF$ **28.** $m\angle EHG$



Animated Geometry at classzone.com

29. ★ **MULTIPLE CHOICE** In parallelogram *ABCD*, $AB = 14$ inches and $BC = 20$ inches. What is the perimeter (in inches) of ▱*ABCD*?

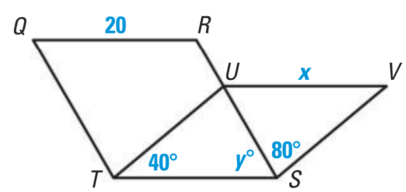
Ⓐ 28 Ⓑ 40 Ⓒ 68 Ⓓ 280

30. xy **ALGEBRA** The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle. Find the measure of each angle.

31. xy **ALGEBRA** The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle. Find the measure of each angle.

32. **ERROR ANALYSIS** In ▱*ABCD*, $m\angle B = 50°$. A student says that $m\angle A = 50°$. *Explain* why this statement is incorrect.

33. **USING A DIAGRAM** In the diagram, *QRST* and *STUV* are parallelograms. Find the values of x and y. *Explain* your reasoning.

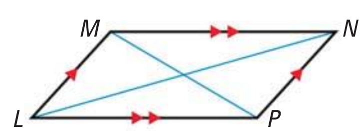


34. **FINDING A PERIMETER** The sides of ▱*MNPQ* are represented by the expressions below. Sketch ▱*MNPQ* and find its perimeter.

$MQ = -2x + 37$ $QP = y + 14$ $NP = x - 5$ $MN = 4y + 5$

35. ★ **SHORT RESPONSE** In *ABCD*, $m\angle B = 124°$, $m\angle A = 66°$, and $m\angle C = 124°$. *Explain* why *ABCD* cannot be a parallelogram.

36. **FINDING ANGLE MEASURES** In ▱*LMNP* shown at the right, $m\angle MLN = 32°$, $m\angle NLP = (x^2)°$, $m\angle MNP = 12x°$, and $\angle MNP$ is an acute angle. Find $m\angle NLP$.



37. **CHALLENGE** Points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of ▱*ABCD*. Find the coordinates of each point that could be vertex *D*. Sketch each possible parallelogram in a separate coordinate plane. *Justify* your answers.

PROBLEM SOLVING

EXAMPLE 2
on p. 517
for Ex. 38

38. AIRPLANE The diagram shows the mechanism for opening the canopy on a small airplane. Two pivot arms attach at four pivot points A, B, C, and D. These points form the vertices of a parallelogram. Find $m\angle D$ when $m\angle C = 40°$. *Explain* your reasoning.

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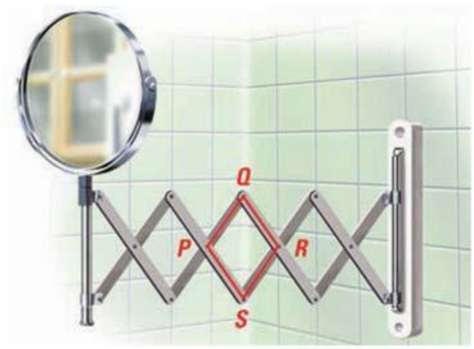
39. MIRROR The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points P, Q, R, and S are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.



a. If $PQ = 3$ inches, find RS.

b. If $m\angle Q = 70°$, what is $m\angle S$?

c. What happens to $m\angle P$ as $m\angle Q$ increases? What happens to QS as $m\angle Q$ decreases? *Explain*.

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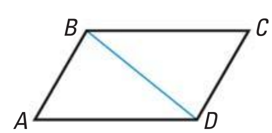
40. USING RATIOS In ▱$LMNO$, the ratio of LM to MN is 4 : 3. Find LM if the perimeter of $LMNO$ is 28.

41. ★ OPEN-ENDED MATH Draw a triangle. Copy the triangle and combine the two triangles to form a quadrilateral. Show that the quadrilateral is a parallelogram. Then show how you can make additional copies of the triangle to form a larger parallelogram that is similar to the first parallelogram. *Justify* your method.

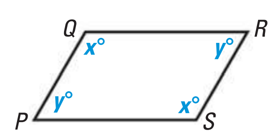
42. PROVING THEOREM 8.4 Use the diagram of quadrilateral $ABCD$ with the auxiliary line segment drawn to write a two-column proof of Theorem 8.4.



GIVEN ▶ $ABCD$ is a parallelogram.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

43. PROVING THEOREM 8.5 Use properties of parallel lines to prove Theorem 8.5.



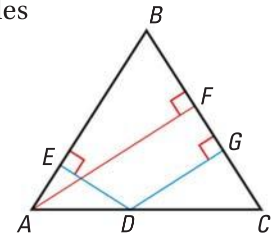
GIVEN ▶ $PQRS$ is a parallelogram.

PROVE ▶ $x° + y° = 180°$

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

44. **PROVING THEOREM 8.6** Theorem 8.6 states that if a quadrilateral is a parallelogram, then its diagonals bisect each other. Write a two-column proof of Theorem 8.6.

45. **CHALLENGE** Suppose you choose a point on the base of an isosceles triangle. You draw segments from that point perpendicular to the legs of the triangle. Prove that the sum of the lengths of those segments is equal to the length of the altitude drawn to one leg.



GIVEN ▶ $\triangle ABC$ is isosceles with base $\overline{AC}$, $\overline{AF}$ is the altitude drawn to $\overline{BC}$, $\overline{DE} \perp \overline{AB}$, $\overline{DG} \perp \overline{BC}$

PROVE ▶ For D anywhere on $\overline{AC}$, $DE + DG = AF$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.3 in Exs. 46–48.

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

46. **Line 1:** (2, 4), (4, 1)
Line 2: (5, 7), (9, 0)

47. **Line 1:** (−6, 7), (−2, 3)
Line 2: (9, −1), (2, 6)

48. **Line 1:** (−3, 0), (−6, 5)
Line 2: (3, −5), (5, −10)

Decide if the side lengths form a triangle. If so, would the triangle be *acute*, *right*, or *obtuse*? *(p. 441)*

49. 9, 13, and 6

50. 10, 12, and 7

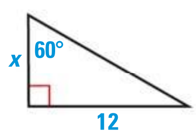
51. 5, 9, and $\sqrt{106}$

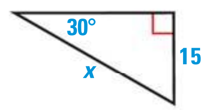
52. 8, 12, and 4

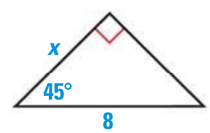
53. 24, 10, and 26

54. 9, 10, and 11

Find the value of *x*. Write your answer in simplest radical form. *(p. 457)*

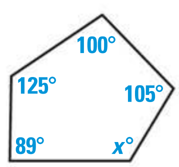
55.


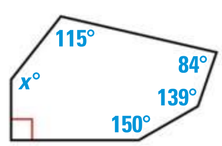
56.


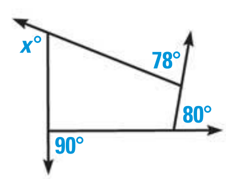
57.


QUIZ *for Lessons 8.1–8.2*

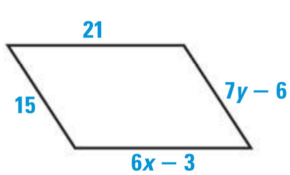
Find the value of *x*. *(p. 507)*

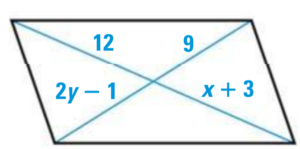
1.


2.


3.


Find the value of each variable in the parallelogram. *(p. 515)*

4.


5.


6.